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STABILITY OF NULL-SPACE CONTROL ALGORITHMS

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ABSTRACT- The paper deals with the stability of control algorithms in extended operational space for redundant robots. We compare the performance of the control algorithm based on minimal-null space projection and pseudo-inverse based projection into the Jacobian null space. We show how the null-space projection affects the stability of the null-space tracking algorithm. The results are verified with the simulation and real implementation on redundant mobile manipulator composed of 3 DOF mobile platform and 4 DOF planar robot arm.

Keywords: Robot Control, Redundant Robots, Jacobian Null-Space

INTRODUCTION

One of the important issues of the new generation of robotic manipulators is the kinematic redundancy. The kinematic redundancy is characterized by extra degrees of freedom with respect to the given motion posed by the assigned primary task. A redundant manipulator has the ability to move the end-effector along the same trajectory using different configurations of the mechanical structure. This provides means for solving sophisticated motion tasks such as avoiding obstacles, avoiding singularities, optimizing manipulability, minimizing joint torques, etc. The result is a significant increase in the dexterity of the system, which is essential to accomplish complex tasks. On the other hand, redundancy has also an important influence on the dynamic behavior of the robotic system. An appropriate control of dynamic properties is essential for higher performance in robotic manipulation. Earlier research in the field of the dynamics of robotic manipulators has been devoted to the dynamics in the joint space. To control the dynamic properties in the joint space different control methodologies have been proposed based on joint space dynamic models [8, 1]. As the next step, methods have been proposed where the control takes place in the task space [7]. These methods include transformations between joint space trajectories and task space trajectories. However, in the case of redundant manipulators these transformations are not unique. Different methodologies have been proposed to resolve the redundancy like optimization of a given performance criteria while satisfying a primary task [14].

To overcome the limitations of control methods based on the joint space dynamics methods Khatib [6] proposed a method for dealing with dynamics and control in the task space. This method enables the description, analysis and control of the robot behavior in the task space, and can also be used for redundant manipulators when the dynamic behavior of the end-effector is of interest. However, for redundant manipulators the end-effector dynamics is only one part of the dynamics of the whole manipulator. The "rest" dynamics represents the dynamics of the internal motion of the manipulator. Hsu and Sastry [4] presented a globally stable controller that tracks the desired null space velocity. They have used Moor-Penrose pseudo-inverse for the redundancy resolution which doesn't decouple the task and null space motion. We have used similar controller with inertia weighted pseudo-inverse and proved stability for high gain of the null space controller [12]. In [10] we proposed modification that assure the stability of the controller for low gains of the null space controller. Similar result was reported in [3]. Another approach was proposed by Chang [2] and latter by Park [16] and Oh [15] based on dynamic decomposition of kinematically redundant manipulators into the task space dynamics and null space dynamics based on a minimally reparametrized homogenous velocity. In their paper they proved stability of the proposed approach and presented experimental results on 3DOF planar manipulator with only one degree of redundancy. However, experimental as well as simulation results have shown certain instability of the overall control algorithm. In our paper we analyse the cause of the instability and propose a robust solution to that problem.

KINEMATICS

Robotic systems under study are n degrees of freedom (DOF) serial manipulators. We consider only the redundant systems which have more DOF than needed to accomplish the task, i.e. the dimension of the joint space n exceeds the dimension of the task space m, $n > m$ and $r = n - m$, is the degree of redundancy. Let the configuration of the manipulator be represented by the vector $\boldsymbol{q}$ of n joint positions, and the end-effector position (and orientation) by m-dimensional vector $\boldsymbol{x}$ of task positions (and orientations). The relation between joint and task velocities is given by the following expression

$$\dot{\boldsymbol{x}} = \mathbf{J}\dot{\boldsymbol{q}} \tag{1}$$

where $\boldsymbol{q}$ is n-dimensional vector of joint positions, $\boldsymbol{x}$ is m-dimensional vector of the end-effector position (and orientation) and $\mathbf{J}$ is the $m\times n$ manipulator Jacobian matrix. The solution of the above equation for $\boldsymbol{q}$ can be given as a sum of particular and homogeneous solution

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_p + \dot{\boldsymbol{q}}_h = \bar{\mathbf{J}}\dot{\boldsymbol{x}} + \bar{\mathbf{N}}\xi \tag{2}$$

where

$$\bar{\mathbf{J}} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1} \tag{3}$$

where $\bar{\mathbf{J}}$ is the weighted pseudo-inverse of $\mathbf{J}$, $\mathbf{W}$ is the weighting matrix, $\bar{\mathbf{N}} = (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})$ is $n\times n$ matrix representing the projection into the null space of $\mathbf{J}$, and ξ is an arbitrary n dimensional vector. $\mathbf{W}$ is the weighting matrix. We will denote this solution as pseudo-inverse based redundancy resolution at velocity level [14]. Since the rank of the null space matrix $\bar{\mathbf{N}}$ is r, the homogeneous solution in Eq. 2 can be presented also in the form

$$\dot{\boldsymbol{q}}_h = \bar{\mathbf{N}}\xi = \mathbf{V}\dot{\boldsymbol{x}}_n \tag{4}$$

where $\mathbf{V}$ is a full column rank $n \times r$ matrix which satisfies the criteria $\mathbf{J}\mathbf{V} = 0$, and $\dot{\boldsymbol{x}}_n$ is an arbitrary r-dimensional minimal null space velocity vector. This approach will be denoted as minimal-null space redundancy resolution at velocity level.

The transformation from joint coordinates to minimal null space velocities is described by

$$\dot{\boldsymbol{x}}_n = \bar{\mathbf{V}}\dot{\boldsymbol{q}} \tag{5}$$

where the pseudo-inverse of $\mathbf{V}$ is defined as

$$\mathbf{V}^+ = \bar{\mathbf{V}} = (\mathbf{V}^T\mathbf{W}\mathbf{V})^{-1}\mathbf{V}^T\mathbf{W} \tag{6}$$

where $\mathbf{W}$ is the weighting matrix. As $\bar{\mathbf{V}}$ maps the joint velocities to the minimal null-space velocities, we denote $\bar{\mathbf{V}}$ as the null-space Jacobian $\mathbf{J}_n$. Using the above formulation we can finally define the extended space $\boldsymbol{x}_e$ as

$$\dot{\boldsymbol{x}}_e = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}}_n \end{bmatrix} = \mathbf{J}_e\dot{\boldsymbol{q}} = \begin{bmatrix} \mathbf{J} \\ \bar{\mathbf{V}} \end{bmatrix}\dot{\boldsymbol{q}} = \begin{bmatrix} \mathbf{J} \\ \mathbf{J}_n \end{bmatrix}\dot{\boldsymbol{q}}, \tag{7}$$

where $\mathbf{J}_e$ is the extended Jacobian. Since $\dot{\boldsymbol{q}} = \bar{\mathbf{J}}\dot{\boldsymbol{x}} + \mathbf{V}\dot{\boldsymbol{x}}_n$, the inverse of $\mathbf{J}_e$ is defined as

$$\mathbf{J}_e^{-1} = \begin{bmatrix} \bar{\mathbf{J}} & \mathbf{V} \end{bmatrix} \tag{8}$$

The null space matrix $\bar{\mathbf{N}}$ and minimal null space matrix $\mathbf{V}$ are related trough

$$\bar{\mathbf{N}} = \mathbf{V}\bar{\mathbf{V}} \tag{9}$$

The above relation is easily verified by postmultiplying Eq. 9 with matrix $\mathbf{V}$. Expanding $\bar{\mathbf{N}}$ and $\bar{\mathbf{V}}$, and considering the equality $\mathbf{J}\mathbf{V} = 0$ yields

$$\begin{aligned} \mathbf{V} - \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}\mathbf{J}\mathbf{V} &= \mathbf{V}(\mathbf{V}^T\mathbf{W}\mathbf{V})^{-1}\mathbf{V}^T\mathbf{W}\mathbf{V} \\ \mathbf{V} &= \mathbf{V} \end{aligned}$$

DYNAMICS

The equation of motion can be reformulated using the extended task space variables

$$\boldsymbol{f}_e = \boldsymbol{\Lambda}_e\ddot{\boldsymbol{x}}_e + \boldsymbol{\mu}_e + \boldsymbol{F}_e \tag{10}$$

where

$$\boldsymbol{f}_e = \mathbf{J}_e^{-T}\boldsymbol{\tau} \tag{11}$$

$$\boldsymbol{\Lambda}_e = \mathbf{J}_e^{-T}\mathbf{H}\mathbf{J}_e^{-1} = \begin{bmatrix} \bar{\mathbf{J}}^T\mathbf{H}\bar{\mathbf{J}} & \bar{\mathbf{J}}^T\mathbf{H}\mathbf{V} \\ \mathbf{V}^T\mathbf{H}\bar{\mathbf{J}} & \mathbf{V}^T\mathbf{H}\mathbf{V} \end{bmatrix} \tag{12}$$

$$\boldsymbol{\mu}_e = \mathbf{J}_e^{-T}\boldsymbol{h} - \boldsymbol{\Lambda}_e\dot{\mathbf{J}}_e\dot{\boldsymbol{q}} \tag{13}$$

$$\boldsymbol{F}_e = \begin{bmatrix} \boldsymbol{F} \\ \mathbf{0} \end{bmatrix} \tag{14}$$

Due to the inertia weighted pseudo-inverse of the Jacobian the off-diagonal elements of $\boldsymbol{\Lambda}_e$ are zero.

IMPEDANCE FORCE CONTROLLER

Let define control vector in extended space as

$$\boldsymbol{f}_c = \boldsymbol{\Lambda}_e\ddot{\boldsymbol{x}}_c + \boldsymbol{\mu}_e + \boldsymbol{F}_e \tag{15}$$

Inserting Eq. 15 into 10 yelds

$$\boldsymbol{\Lambda}_e(\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}}_e) = 0 \tag{16}$$

$\ddot{\boldsymbol{x}}_c$ denotes control vector in the form

$$\ddot{\boldsymbol{x}}_c = \begin{bmatrix} \ddot{\boldsymbol{x}}_d + \mathbf{K}_v\dot{\boldsymbol{e}} + \mathbf{K}_p\boldsymbol{e} \\ \ddot{\boldsymbol{x}}_{nd} + \mathbf{K}_n\dot{\boldsymbol{e}}_n \end{bmatrix} \tag{17}$$

, where $\boldsymbol{e}$ is tracking error, subscript n denotes null space and subscript d denotes desired signal.

Again, by selecting $\mathbf{W} = \mathbf{H}$ in Eq. 6 the off diagonal elements of extended inertia matrix are zero. Eq.17 is thus decoupled into two equations

$$\bar{\mathbf{J}}^T\mathbf{H}\bar{\mathbf{J}}(\ddot{\boldsymbol{e}} + \mathbf{K}_v\dot{\boldsymbol{e}} + \mathbf{K}_p\boldsymbol{e}) = 0$$

$$\mathbf{V}^T\mathbf{H}\mathbf{V}(\ddot{\boldsymbol{e}}_n+\mathbf{K}_n\dot{\boldsymbol{e}}_n)=0 \tag{18}$$

Since both matrices $\bar{\mathbf{J}}^T\mathbf{H}\bar{\mathbf{J}}$ and is $\mathbf{V}^T\mathbf{H}\mathbf{V}$ are positive definite, it follows $\ddot{\boldsymbol{e}}+\mathbf{K}_v\dot{\boldsymbol{e}}+\mathbf{K}_p\boldsymbol{e}=0$ and $\ddot{\boldsymbol{e}}_n+\mathbf{K}_n\dot{\boldsymbol{e}}_n=0$. Error equation shows the main advantage of the minimal-null space approach. Only minimal null-space approach assures desired dynamic behaviour in the null-space, which can not be guarantied for pseudo-inverse based controller. The reason for this is in the existence of the pseudoinverse of minimal null-space transformation matrix $\mathbf{V}$. On contrary, matrix $\bar{\mathbf{N}}$ is rank deficient and pseudoinverse of $\bar{\mathbf{N}}$ does not exists.

Joint space control law can be derived from Eq. 15 and is

$$\begin{aligned}\tau_c = \;& \mathbf{H}\bar{\mathbf{J}}(\ddot{\boldsymbol{x}}_d+\mathbf{K}_v\dot{\boldsymbol{e}}+\mathbf{K}_p\boldsymbol{e}-\dot{\bar{\mathbf{J}}}\dot{\boldsymbol{q}})+\\ & \mathbf{H}\mathbf{V}(\ddot{\boldsymbol{x}}_n-\dot{\mathbf{V}}\dot{\boldsymbol{q}})+\boldsymbol{h}+\mathbf{J}^T\boldsymbol{F}\end{aligned} \tag{19}$$

First term corresponds to the task-space control, second to the null-space control and third and fourth to the compensation of non-linear system dynamics and external force respectevely.

MINIMAL NULL SPACE CALCULATION

There is an infinite number of possible null space transformations. In [11] we have shown that the null space motion depends only on criteria function and is therefore independent on the selection of the null space transformation $\mathbf{V}$. On the other hand, the numerical stability of the control algorithm is subjected on the selection of $\mathbf{V}$. Namely, representation of the null space with the base vectors is not unique. There is an infinite number of orthonormal basis vectors $\mathbf{V}$ that describe the same null space. For good control there is necessary to obtain smooth continuous solution of $\mathbf{V}$ during the execution of the robot's task.

There are several methods how to obtain $\mathbf{V}$. Method proposed by Park [16] uses singular value decomposition (SVD) of $\mathbf{J}$ or, alternatively, $\mathbf{J}^T\mathbf{J}$. Singular value decomposition or $\mathbf{J}$ yields

$$\mathbf{U}\boldsymbol{\Sigma}\mathbf{Z}^T=\mathbf{J} \tag{20}$$

$\boldsymbol{\Sigma}$ is the diagonal matrix of m non-zero eigenvalues s and $n-m$ zero eigenvalues of $\mathbf{J}$. The corresponding matrices $\mathbf{Z}$ have form $\mathbf{Z}=\begin{bmatrix}\mathbf{R}\\ \hdashline \mathbf{V}\end{bmatrix}$ and $\mathbf{U}^T=[\mathbf{Q}|\mathbf{V}^T]$. Since matrices $\mathbf{U}$ and $\mathbf{Z}$ are unitary, it follows

$$\mathbf{U}^T\mathbf{J}\mathbf{Z}=\Sigma$$

and

$$\begin{bmatrix}\mathbf{Q} \,\vdots\, \mathbf{V}^T\end{bmatrix}\mathbf{J}\begin{bmatrix}\mathbf{R}\\ \hdashline \mathbf{V}\end{bmatrix}=\left[\begin{array}{cccc|c} s_1 & & & & \\ & s_2 & & & \mathbf{0}\\ & & \ddots & & \\ & & & s_m & \\ \hline & & \mathbf{0} & & \mathbf{0}\end{array}\right] \tag{21}$$

Obviously, $\mathbf{QJV}=\mathbf{0},\mathbf{V}^T\mathbf{JR}=\mathbf{0}$, and, since $\mathbf{Q}$ and $\mathbf{R}$ are non-zero matrices, $\mathbf{JV}=\mathbf{0},\mathbf{V}^T\mathbf{J}=\mathbf{0}$ and thus sub-matrix $\mathbf{V}$ forms null-space of $\mathbf{J}$. Unfortunately, matrix $\mathbf{V}$ is not unique. There is an infinite number of orthonormal basis vectors $\mathbf{V}$ that describe the same null space.

The most popular technique for computing the SVD is Golub-Reunsch algorithm and is available in many linear algebra software packages. It is regarded as the most efficient and numerically stable technique for computing the SVD of an arbirtary matrix. Unfortunately, it does not assures the continuous solutions. For example, if matrix $\mathbf{J}$ changes contiguously, this not implies that also matrices $\mathbf{U}$, $\boldsymbol{\Sigma}$ and $\mathbf{Z}$ will change continuously. We will demonstrate this effect with the simulation of 100 instances of the kinematics of the 4-DOF planar manipulator with links of equal length. The $\mathbf{V}$ matrix was calculated using MATLAB function `null`, which is based on SVD calculation using Golub-Reunsch algorithm. The constant null space velocity $\boldsymbol{x}_n=[10,0]$ was applied to initiate null-space motion. At $\boldsymbol{q}=[-2.7546,-0.4373,1.0346,0.0095]$ null-space motion stops. The reason is that solution of $\mathbf{V}$ alternates between two set of values, which is demonstrated in fig. 1 Discontinuity of elements in $\mathbf{V}$ causes numerical instability of the control algorithm. Namely, control algorithm 19 requires $\dot{\mathbf{V}}$, which has to be differentiated numerically. Discontinuity of $\mathbf{V}$ results in an unbounded control signal.

The solution of the above problem is SVD algorithm based on Givens rotations. The approach was reviewed by Maciejewski and Klein [9] as an algorithm, more suited to take advantage of incremental perturbations and parallel architectures. For our purpose we do not need to calculate all matrices of SVD. We need only matrix $\mathbf{Z}$, which orthogonalizes the columns of $\mathbf{J}$. This matrix is usually formed as a product of Givens rotations, each of which orthogonalize two columns. Considering the current i-th and j-th colums of $\mathbf{J}$, a multiplication by Givens rotation results in new colums

$$\begin{aligned}\mathbf{J}_i^* &= \mathbf{J}_i\cos(\theta)+\mathbf{J}_j\sin(\theta)\\ \mathbf{J}_j^* &= \mathbf{J}_j\cos(\theta)-\mathbf{J}_i\sin(\theta)\end{aligned} \tag{22}$$

with constraint $\mathbf{J}_i^*\mathbf{J}_j^*=0$. The terms in the Givens rotation matrix which ortogonalizes $\mathbf{J}$ can be found in

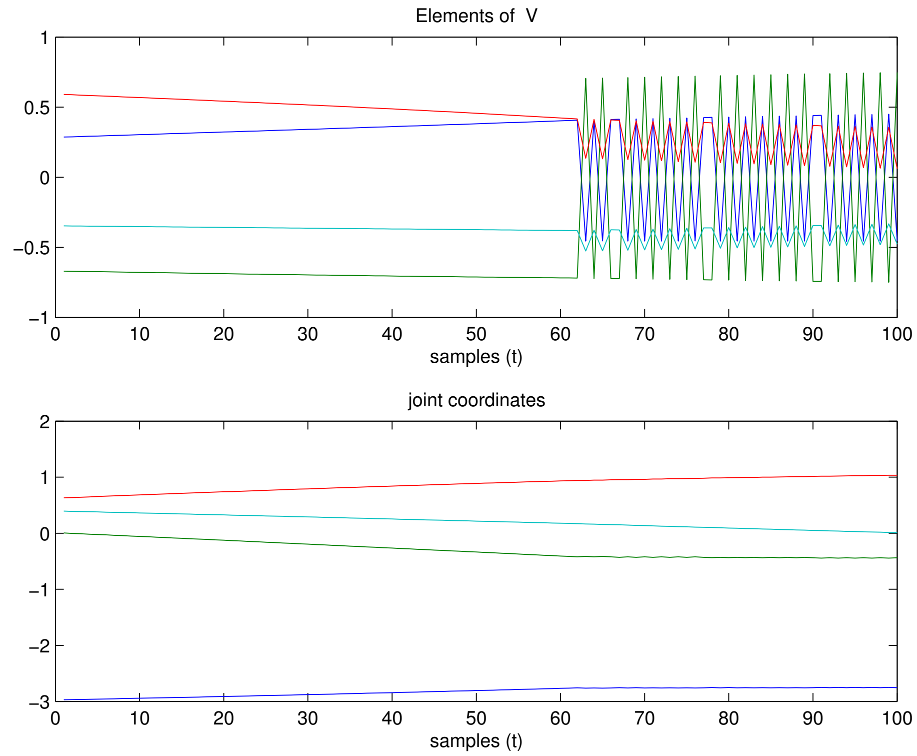

Figure 1. elements of the 1st columns of $\mathbf{V}$ and joint angles

[9]. However, orthogonalisation can not be acheived in single sweep, in general we need multiple sweeps, but the algorithm converges [9]. But perhaps the most useful property of the algorithm is the ability to use perturbed initial value of matrix $\mathbf{Z}$. The more orthogonal are the columns of $\mathbf{JZ}$, fewer are the number of sweeps required for convergence, and, which is more important in our case, the solutions are contiguous. If one considers the current $\mathbf{J}$ to be a perturbation of the previous $\mathbf{J}$, $\mathbf{J}(t+\delta t) = \mathbf{J}(t) + \delta\mathbf{J}(t)$, then the matrix $\mathbf{J}(t+\delta t)\mathbf{Z}(t)$ will have nearly orthogonal columns. Since control of the manipulator consists of subsequent calculations of Eq. 19, we can use the solution of the $\mathbf{Z}$ from the previous step, which improves the convergence of the algorithm, reduces computational burden, and, which is most important, assures contiguous solution of the Jacobian null space matrix $\mathbf{V}$.

NULL SPACE MOTION OPTIMIZATION AND OBSTACLE AVOIDANCE

The desired extended null space velocities which minimize the given criteria can be obtained using weighted gradient optimization procedure [13]

$$\dot{\boldsymbol{x}}_n = (\mathbf{V}^T\mathbf{H}\mathbf{V})^{-1}\mathbf{V}^T\frac{\partial p}{\partial \boldsymbol{q}}k_o, \qquad (23)$$

which assures the best optimization step in the case of inertia weighted pseudo-inverse. k_o is a negative constant and defines the optimization step.

The force and the position tracking are usually of the highest priority for a force controlled robot. The selection of the sub-tasks with lower priority depends on the specific application. However, the collision avoidance is of great importance in most application of redundant robot system, since it is very difficult to predict the path of all links. In most cases the motion is not guaranteed to be conservative, therefore one collision free task cycle does not imply next collision free cycle.

Following the idea of the obstacle avoidance using the potential field [5] we define the cost function $\boldsymbol{p} = \frac{1}{2}\mathbf{E}d_0^2$, where $\mathbf{E}$ is an $l \times l$ rotation matrix describing the direction of an artificial potential field pointing from the obstacle, l is the dimension of the position sub-space and d_0 is the shortest distance between obstacle and the robot body. In our case the desired objective is fulfilled if the imaginary force is applied only on robot joints. In this case we can obtain cost function gradient in simple form as

$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{q}} = \mathbf{E}(\boldsymbol{d}_1\mathbf{J}^{0,1} + \boldsymbol{d}_2\mathbf{J}^{0,2} + \ldots.. + \boldsymbol{d}_{n-1}\mathbf{J}^{0,n-1}), \qquad (24)$$

where $\boldsymbol{d}_i$ is the vector of shortest distances between the i-th joint and the obstacle and $\mathbf{J}^{0,i}$ denotes Jacobian matrices between base (the first index in the superscript) and ith joint (the second index in the superscript) regarding the robot positions only.

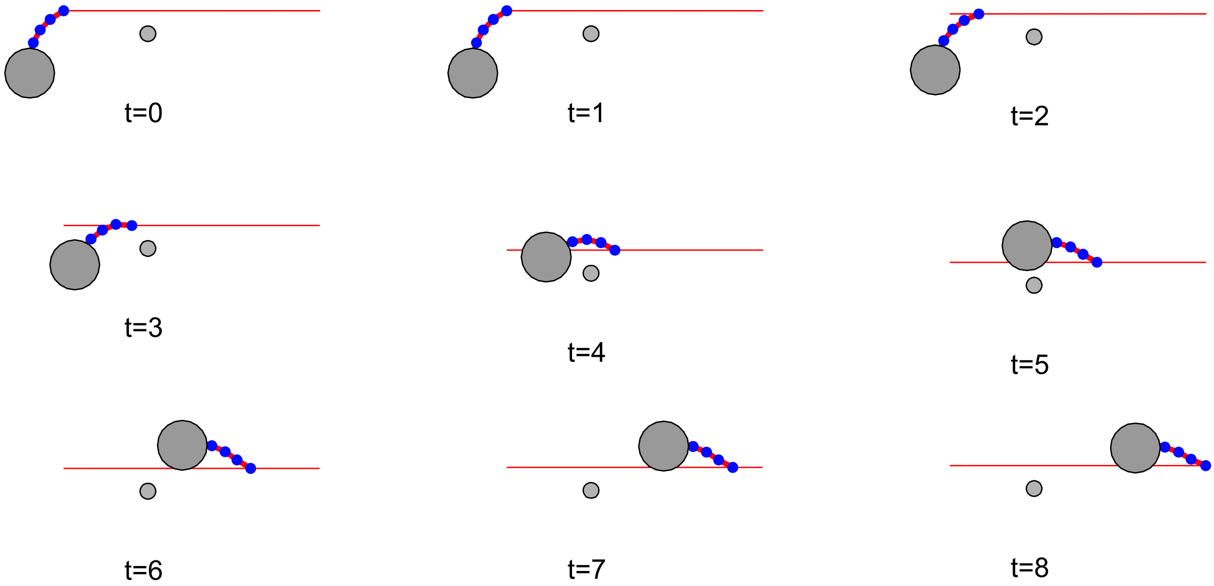

Figure 3. Time instances of the simulated task

EXPERIMENTS

The experimental setup consists of 4-DOF planar robot arm with all segments of equal length $0.25m$, mounted on the holonomous mobile platform with 3-DOF. Whole setup is presented in Fig. 2. The robot arm had no limits in joint angles. All AC brushless motors were located in the robot base in order to obtain lightweight links. The robot arm gear ratio was 6, thus the coupled dynamics of the robot arm was not negligible. On the other hand, mobile platform is much heavier than the robot arm, therefore the influence of dynamic coupling between the mobile platform and robot arm is negligible on the mobile platform side, but not on the robot arm side. The robot controller consists of a Pentium II 360 MHz industrial computer. The proposed control algorithm was realized on Simulink and compiled using Simulink Real Time Workshop and Planar Manipulator Toolbox. The primary task of the manipulator was to track the line. The desired speed was $0.45m/s$. There was an obstacle in the robot work-space, as shown in Fig. 3. The secondary subtask was obstacle avoidance. Regarding the given task, the degree of redundancy was 5. High degree of redundancy combined with mobile platform requires careful selection of the secondary tasks. Tasks such as manipulability optimization or torque optimization will always lock the robot arm into the optimal position and the task motion will be performed with mobile base only. This is not the case when the obstacle avoidance is used. The top view of the simulated motion of the mobile manipulator is shown in Fig. 3.

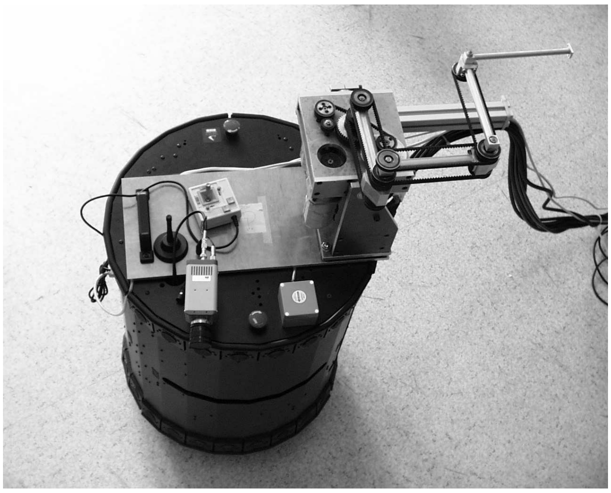

Figure 2. experimental mobile robot

CONCLUSION

The paper considers the stability of the control algorithms for redundant robots using minimal null-space force. In the paper it was shown that algorithm based on SVD based on Golub-Reunsch algorithm causes instability of the overall control scheme. We proposed the solution based on SVD calculation using Givens rotations. The proposed control scheme was tested on simulation of 7 D.O.F mobile manipulator system. The primary task was trajectory tracking, while avoiding the obstacles as a secondary subtask. The results shows good numerical stability and shorter computational cycle comparing to the SVD based on Golub-Reunsch algorithm. Currently, we are preparing experiments with real robot interacting with the environment.

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